Heat transfer from turbulent water flow in a tube to a cooled isothermal wall

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A device was constructed to establish a constant wall temperature for the isothermal cooling of the turbulent flow of water in a tube. The flow was fully developed hydrodynamically but developing thermally. Data obtained were used to derive an equation to predict the Nusselt number using the method of least squares. An existing equation for developing flow was tested to determine how well it fit the data. The derived equation of this study has a correlation coefficient of 0.877 and a standard deviation of 19.0.

Keywords: convection; turbulent flow; isothermal tube wall

Introduction

The purpose of this study was to examine the equations for determination of overall heat transfer coefficients in the cooling process of a fluid with variable properties. The heat transfer occurred in a hydrodynamically developed, thermally developing turbulent flow of water in a tube with walls at constant temperature.

The earlier correlations of the general form

$$\mathrm{Nu} = C\,\mathrm{Re}^a\mathrm{Pr}^n \tag{1}$$

are widely preferred for this type of problem due to their simplicity. They are, however, very conservative at high Reynolds numbers and intermediate Prandtl numbers. To use equations of the above form, the designer must be liberal in determining safety factors. These equations also begin to fail as fluid properties become variable. Studies with the Dittus-Boelter equation[1] in particular were conducted by varying the wall to bulk-temperature differences and extrapolating the results to zero-temperature difference for the constant-property case. Sieder and Tate[2] considered the variable-property-data problem in their analysis of oils and formulated an equation with a viscosity correction. Recent studies using more developed analytical methods contain equations in which turbulent heat transfer in the laminar sublayer, the buffer zone, and the turbulent core is considered. Petukhov[3], using more accurate relationships for velocity distributions and eddy diffusivities, applied the method of successive approximations in correlating variable-fluid-property data with the result that accuracy was within 10%. Sleicher and Rouse[4] obtained an expression with similar results and an accuracy of 20%. Most of these studies have dealt only with the case of fluid heating, both with a constant heat flux and with constant wall temperature.

The analysis of heat transfer in turbulent pipe flow for forced convection by similarity methods was first performed by Nusselt[5]. The method yielded a final equation form that related the Nusselt number to the Reynolds number and the Prandtl number:

$$\mathrm{Nu}_D = f_1(\mathrm{Re})f_2(\mathrm{Pr}) \tag{2}$$

Nusselt's original assumption of constant fluid properties introduced difficulties due to the temperature-dependent properties of real fluids. Also assumed is that the heat flow direction is inconsequential, but the modes of heating and cooling affect properties inversely. One method of solution is to solve for the coefficients in Nusselt's equation applied to the two separate cases. Another method relies on specification of a temperature for evaluation of properties. A common practice is to use the bulk temperature for evaluation of properties.

Other assumptions made in the problem formulation are steady flow, steady-state heat transfer, and fully developed conditions. For shorter duct lengths where fully developed conditions do not exist, corrections are needed.

The theoretical solution to heat transfer in turbulent pipe flow begins by dividing the flow into three layers: the viscous sublayer, the buffer zone, and the turbulent core. Numerical methods allow for use of more accurate relationships for the velocity distribution, the eddy diffusivity, and the eddy viscosity. The solutions predicted by von Karman[6] are applicable except at higher Prandtl numbers (greater than 10) for conditions near the wall.

The heat transfer correlation for variable fluid properties determined by Sleicher and Rouse[4] is based upon the constant-fluid-property correlation of Notter and Sleicher[7]. The choice of eddy viscosity was taken from a previous study, and the turbulent Prandtl number was taken to be 1.3. The method of solution consisted of numerically solving for the lower eigenvalues and constants in the temperature distribution. For a developing flow, entrance effects must be considered. Entrance effects can be accounted for by a correction of the form[8]

$$\frac{\overline{\mathrm{Nu}}_D(L)}{\overline{\mathrm{Nu}}_D(L=\infty)} = 1 + C_1\left(\frac{L}{D}\right)^{C_2} \tag{3}$$

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where C_1 and C_2 are factors that depend on the Reynolds and Prandtl numbers. It is believed that use of this procedure cannot be generally accurate because of this dependence[9].

Analyses have been performed for the problem where the velocity and temperature profiles develop simultaneously[10–13]. Many of these models use integral methods, but reliance is made primarily on experimental results due to the complexities encountered for each case. On the basis of experiments conducted with fluids such as air, corrections similar to that of Equation 3 have been proposed[14]. There is enough scatter in the measurements such that the constant-wall-temperature case is indistinguishable from the constant-wall-flux case in the entrance region correction. For developing flows, Nusselt[5] recommended the correlation

$$\overline{\mathrm{Nu}} = 0.036\,\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}(L/D)^{0.055} \tag{4}$$

where $10 < L/D < 400$. Nusselt assumed incompressible fluid flow with gravity forces negligible compared to pressure forces. He also assumed constant fluid properties, steady fluid flow, steady heat flow, and that the heat flow direction was immaterial. The constant-property assumption and the assumption that heat flow direction is immaterial deviate somewhat from the real case.

All previous studies found in the literature involving heat transfer in duct flow deal with the heating problem. That is, the fluid was heated with a constant wall flux or a constant wall temperature. In this study, we deal with the cooling problem where the fluid is in contact with a cooler constant-temperature tube surface. Further, as our model we elect to use an equation of the form given in Equation 1. An equation will be developed to describe heat transfer from the turbulent developing flow of water to a cooled isothermal wall. Nusselt's recommended correlation (Equation 4) will be compared to the experimental results obtained.

Experimental apparatus and procedure

The experimental test section consists of a double-pipe heat exchanger in which water (the fluid of interest) flows through the inner tube and the coolant (freon-12) flows in the annulus in the opposite direction. Isothermal cooling is maintained by constant-temperature evaporation of freon-12 in the evaporator stage of a conventional air conditioner.

Apparatus

Figure 1 shows a schematic of the apparatus. It contains two fluid circuits: that of water and that of freon-12. Tap water is continuously circulated through a $\frac{3}{8}$-in-ID, $\frac{1}{2}$-in-OD copper tube, used throughout for conveying the water. A Dayton $\frac{1}{2}$-hp stainless steel centrifugal pump (model 1P799B) draws water from a 15-gal Nalgene tank and circulates water through the circuit. The water is routed through a 3.5-gal/min variable-area flowmeter (model RMC-143-SSV) manufactured by Dwyer Instruments, Inc., and then through the inner tube of three double-pipe heat exchangers. The water can be heated at this stage at the option of the user. The water is then routed vertically to a 45-in-long tube located just upstream of three more double-pipe heat exchangers. (The 45-in entrance to the test section corresponds to an L/D value of 119.) These three exchangers are test sections. In this study, only the first exchanger was used to obtain data. After leaving this part of the apparatus, the water enters an exit length of 12 in and is piped back to the tank. Upstream of the tank is placed an air-cooled cross-flow heat exchanger (Morril Electric model SP-A5EM) for further cooling if desired.

The refrigerant circuit consists of copper tubes of various sizes and contains the basic conventional air conditioner components. The compressor is rated at 12,000 BTU/h and is manufactured by Tecumseh (model R12-2W). Freon-12 leaving the compressor is routed to a condenser section. The condenser is made up of two components. The first is a cross-flow heat exchanger used to dissipate unwanted energy or heat. The second is a system similar to the test sections (three double-pipe heat exchangers) designed so as to use some of the rejected heat in the cycle to heat (if desired) the water at the inlet. After leaving the condenser area, the freon-12 passes through a rotameter, an expansion valve, and three regulating cutoff vales. The cutoff valves control the flow of refrigerant to each test section. The valves were used to reglate the flow of freon-12 to ensure that the temperature drops across the exchangers were (practically) zero. A zero-degree superheat is required in the experiments, and care was exercised in obtaining it. After leaving the evaporator (i.e., the test section double-pipe heat exchangers), the freon-12 is routed back to the compressor.

Additional heat can be added to the water by a 120-V variac-regulated Briskheat flexible electric heating tape (Thermolyne Corp.) wound around the tube just downstream of the condensers and from an immersion heater placed in the sump

Notation

c_p	Specific heat of fluid, J/(kg-K)
D	Inside diameter of duct, m
$\bar{h}_L$	Average heat transfer coefficient for the inside tube surface, W/(m²-K)
$\bar{h}_0$	Average heat transfer coefficient for the outside tube surface, W/(m²-K)
ID	Inside diameter of tube, m
k	Thermal conductivity, W/(m-K)
L	Tube length, m
Nu	Nusselt number
$\overline{\mathrm{Nu}}_D$	Overall or average Nusselt number using diameter as characteristic length
OD	Outside diameter of tube, m
Pr	Prandtl number
Q	Volume flow rate, m³/s
q	Heat flow rate, W
Re	Reynolds number
T	Temperature of tube fluid, °C
t	Temperature of annulus fluid, °C
U_0	Overall heat transfer coefficient for double pipe heat exchanger based on outside surface area of tube, W/(m²-K)
V	Average velocity of fluid in the duct, m/s
z_h	Hydrodynamic entrance length, m
z_t	Thermal entrance length, m
μ	Absolute viscosity, N-s/m²
ν	Kinematic viscosity, m²/s
ρ	Density, kg/m³

Subscripts

1	Inlet
2	Outlet
Cu	Copper tube
wi	Tube inside wall surface
wo	Tube outside wall surface

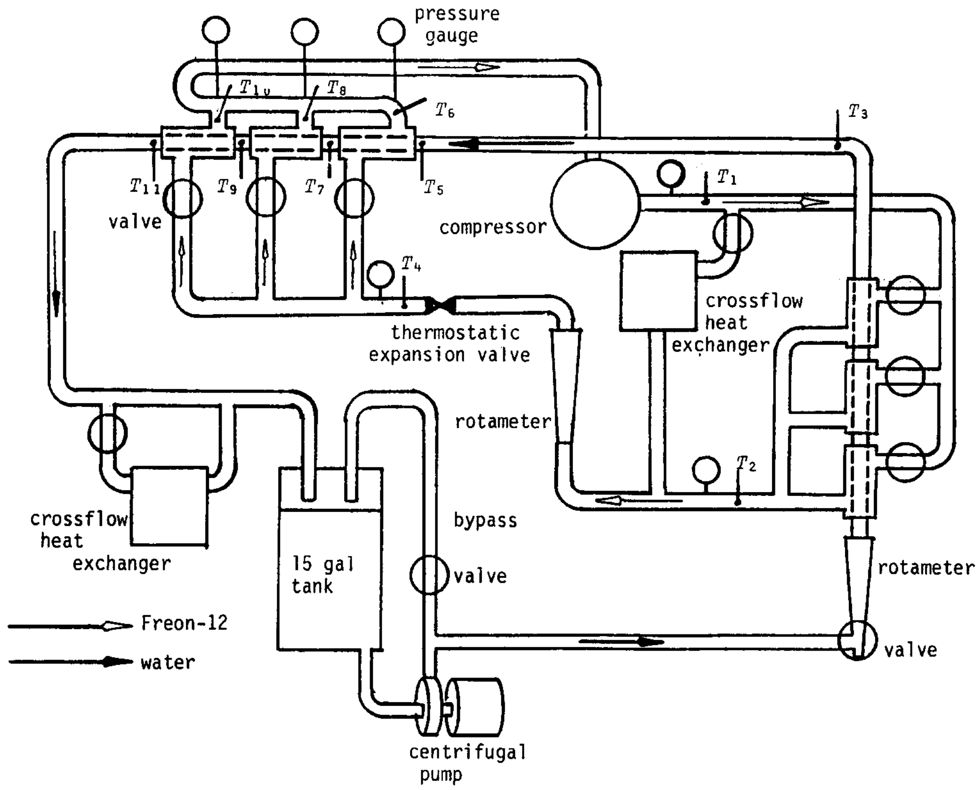

Figure 1 Schematic of the experimental device

tank. The entrance and test sections are lagged with fiberglass insulation.

Chromel-alumel thermocouples were stationed at various locations to provide readings of water inlet temperature to the test section (T_5 of Figure 1), water outlet temperature (T_7), freon-12 inlet temperature (T_4), and outlet temperature (T_6). Thermocouples were connected to a Molytek microprocessor-based multipoint recorder (model 2700). Pressure gauges were also located throughout the system.

Procedure

Both circuits were started up, and freon temperatures before and after the evaporator (T_4 and T_6, T_8, T_{10}, respectively, of Figure 1) were adjusted by the regulating valves to isothermal conditions. A water flow was chosen with a specified inlet temperature. Freon and water temperatures (T_4, T_6, T_5, and T_7) were monitored in the test section until steady-state conditions were established (data points were continuously plotted by the recorder). When steady-state conditions were achieved, the inlet and outlet temperatures of the water and freon, the water flowrate, and the inlet and outlet freon pressures were recorded. The valve was then adjusted for varying water flows with readjustment of the freon temperatures each time to ensure that proper bondary conditions were met. The above procedure was repeated for a wide range of water temperatures. The time required for collection of one set of data ranged from 10 min to several hours.

Results and analysis

The range of variables is shown in Table 1. Table 2 gives the raw data obtained in this experiment. As indicated, the volume flowrate of water varied from 0.85 to 2.2 gal/min. Water inlet temperatures varied from 18.5 to 47.85°C. The freon-12 temperatures should have remained constant for any run, but this was not always possible. Therefore, runs that had freon-12 temperature differences of greater than 1°C were rejected, so there was confidence in maintaining isothermal conditions in the test section annulus. Water temperature differences of less than 1°C were rejected because of instrument sensitivity.

Table 1 Range of variables covered in tests

Item	Range
Volume flow rate, m^3/s (gal/min)	5.36–13.88 ($\times 10^5$) (0.85–2.20)
Water inlet temperature, °C (°F)	18.5–47.85 (65.3–118.13)
Water temperature differences, °C (°F)	1.4–7.40 (2.8–12.0)
Average freon-12 temperatures, °C (°F)	−4.37–13.80 (24.13–56.84)
Bulk to wall-temperature differences, °C (°F)	11.95–34.35 (21.51–61.83)
Bulk Reynolds numbers	10,250–31,180
Bulk Prandtl numbers	3.90–7.50
Bulk Nusselt numbers	82.0–216

It was desired to calculate the Nusselt and Reynolds numbers. The Prandtl numbers were obtained from property data tables. The Reynolds number was calculated from the equation

$$\mathrm{Re} = \rho V D/\mu g_c = VD/\nu$$

For $V = 4Q/\pi D^2$,

$$\mathrm{Re} = 4Q/\pi D \nu$$

It was necessary to determine the inside wall convection coefficient (i.e., between the water and the copper tube of the test

Table 2 Experimental data: temperatures and flowrates

	Water			Freon-12	
Test number	Inlet temperature °C (°F)	Outlet temperature °C (°F)	Volume flowrate $m^3/s \times 10^5$ (gal/min)	Inlet temperature °C (°F)	Outlet temperature °C (°F)
1	43.40 (110.12)	36.75 (98.15)	5.52 (0.875)	5.6 (42.08)	5.9 (42.62)
2	18.50 (65.30)	16.70 (62.06)	8.20 (1.30)	0.2 (32.26)	0.0 (32.00)
3	39.70 (103.46)	34.4 (93.92)	6.15 (0.975)	8.6 (47.48)	8.4 (47.12)
4	25.3 (77.54)	22.8 (73.04)	7.89 (1.25)	7.4 (45.32)	7.05 (44.69)
5	45.65 (114.17)	40.1 (104.18)	6.15 (0.975)	9.2 (48.56)	8.8 (47.84)
6	48.4 (119.12)	41.0 (105.80)	6.15 (0.975)	10.9 (51.62)	11.1 (51.98)
7	42.6 (108.68)	37.05 (98.69)	6.62 (1.05)	4.0 (39.20)	4.2 (39.56)
8	21.2 (70.16)	19.08 (66.34)	9.46 (1.50)	1.1 (33.98)	0.9 (33.62)
9	26.2 (79.16)	23.95 (75.11)	8.99 (1.425)	7.0 (44.60)	6.8 (44.24)
10	39.9 (103.82)	35.15 (95.27)	7.41 (1.175)	8.9 (47.12)	9.2 (48.56)
11	44.3 (111.74)	39.15 (102.47)	6.94 (1.10)	7.0 (44.60)	6.9 (44.42)
12	32.2 (89.96)	30.8 (87.44)	8.20 (1.30)	18.5 (65.30)	19.2 (66.56)
13	30.65 (87.17)	28.45 (83.21)	8.99 (1.425)	12.7 (54.86)	12.5 (54.5)
14	29.55 (85.19)	27.5 (81.50)	9.30 (1.475)	11.8 (53.24)	11.6 (52.88)
15	46.10 (114.98)	41.1 (105.98)	7.41 (1.175)	7.7 (45.88)	7.7 (45.86)
16	44.6 (112.28)	40.1 (104.18)	8.20 (1.30)	7.3 (45.14)	7.5 (45.50)
17	47.85 (118.13)	42.75 (108.95)	8.20 (1.30)	12.1 (53.78)	12.1 (53.78)
18	39.9 (103.82)	36.0 (96.80)	9.30 (1.475)	9.4 (48.92)	10.0 (50.00)
19	25.2 (77.36)	23.45 (74.21)	11.99 (1.90)	7.7 (45.86)	7.5 (45.50)
20	46.3 (115.34)	41.75 (107.15)	8.52 (1.35)	7.6 (45.68)	7.8 (46.04)
21	44.95 (112.91)	40.9 (105.62)	9.30 (1.475)	7.9 (46.22)	8.1 (46.58)
22	42.65 (108.77)	38.5 (101.30)	10.09 (1.60)	4.6 (40.28)	4.8 (40.64)
23	29.25 (84.65)	27.6 (81.68)	12.77 (2.025)	11.5 (52.70)	11.3 (52.34)
24	39.3 (102.74)	35.9 (96.62)	11.36 (1.80)	10.7 (51.26)	10.0 (50.00)
25	46.6 (115.88)	42.7 (108.86)	10.09 (1.60)	8.1 (46.58)	8.4 (47.12)
26	30.4 (86.72)	38.85 (83.93)	13.56 (2.15)	13.9 (57.02)	13.7 (56.66)
27	45.30 (113.54)	41.6 (106.88)	10.72 (1.70)	8.1 (46.58)	8.4 (47.12)
28	46.45 (115.61)	42.9 (109.22)	11.67 (1.85)	8.8 (47.84)	8.4 (47.12)
29	45.5 (113.90)	42.15 (107.87)	12.30 (1.95)	8.3 (46.94)	8.7 (47.66)
30	39.3 (102.74)	36.35 (97.43)	14.04 (2.225)	11.8 (53.24)	11.2 (52.16)
31	46.45 (115.61)	43.2 (109.76)	13.25 (2.10)	9.0 (47.66)	8.7 (47.66)
32	45.3 (113.54)	42.35 (108.12)	14.35 (2.275)	8.4 (48.02)	8.9 (48.02)

section) to calculate the Nusselt number. This was not a straightforward calculation, and a number of analyses were tried. A successful technique was developed and now follows.

Figure 2 is a sketch of a double-pipe heat exchanger as used in this experiment. Known are the volume flowrate of the tube fluid and the fluid inlet and outlet temperatures: T_1 and T_2 for the tube fluid inlet and outlet, respectively, and t_1 and t_2 for the annular fluid. For this case, $t_1 = t_2 = t$ under isothermal cooling conditions. The wall temperatures are T_{wi} and T_{wo}; these are for the inside and outside tube surfaces, respectively. Also, for this configuration,

$$T_1 > T_2 > T_{wi} > T_{wo} > t$$

The heat transferred for the isothermal problem is

$$q = \bar{h}_L \pi(\mathrm{ID})L \frac{(T_1 - T_{wi}) - (T_2 - T_{wi})}{\ln((T_1 - T_{wi})/(T_2 - T_{wi}))} \tag{5}$$

The energy transferred to the tube fluid is

$$q = \dot{m}c_p(T_1 - T_2) = \rho Q c_p(T_1 - T_2) \tag{6}$$

where properties are evaluated at $(T_1 + T_2)/2$. A third equation for the heat transferred in terms of an overall heat transfer coefficient U_0 based on the outside surface area of the tube is

$$q = U_0 \pi(\mathrm{OD})L \frac{(T_1 - t) - (T_2 - t)}{\ln((T_1 - t)/(T_2 - t))} \tag{7}$$

where

$$\frac{1}{U_0} = \frac{\mathrm{OD}}{\mathrm{ID}} \frac{1}{\bar{h}_L} + \frac{\mathrm{OD}}{2k_{\mathrm{Cu}}} \ln \frac{\mathrm{OD}}{\mathrm{ID}} + \frac{1}{\bar{h}_0} \tag{8}$$

A fourth equation for the heat transferred is

$$q = \left(\frac{\mathrm{OD}}{2k_{\mathrm{Cu}}} \ln \frac{\mathrm{OD}}{\mathrm{ID}} + \frac{1}{\bar{h}_0}\right)^{-1} \pi(\mathrm{OD})L(T_{wi} - t) \tag{9}$$

In examining Equations 5–9, we have ID, OD, T_1, T_2, t, $\underline{k}$, k_{Cu}, ρ, Q, $\underline{c}_p$, and L as known quantities for any run. We have $\bar{h}_L$, T_{wi}, $\bar{h}_0$, and U_0 as unknown quantities. For all runs, these equations were solved simultaneously to obtain h_L needed to calculate the Nusselt number Nu $(= \bar{h}_L D/k)$.

The entrance length required for fully developed flow conditions to exist was calculated with

$$z_h = 4.4D(\mathrm{Re})^{1/6} \quad \text{(hydrodynamic)} \tag{10a}$$

$$z_t = 0.055D \,\mathrm{Re}\,\mathrm{Pr} \quad \text{(thermal)} \tag{10b}$$

Sample calculation

For the first run listed in Table 2, we have $Q = 0.875$ gpm $= 5.52 \times 10^{-5}\ \mathrm{m^3/s}$, $T_1 = 43.40°\mathrm{C}$, $T_2 = 36.75°\mathrm{C}$, and $t = (5.6 + 5.9)/2 = 5.75°\mathrm{C}$. A computer program was written to derive an equation for each of the properties of water (i.e., ρ, c_p, k, ν) as a function of temperature. For this example, $(T_1 + T_2)/2 =$

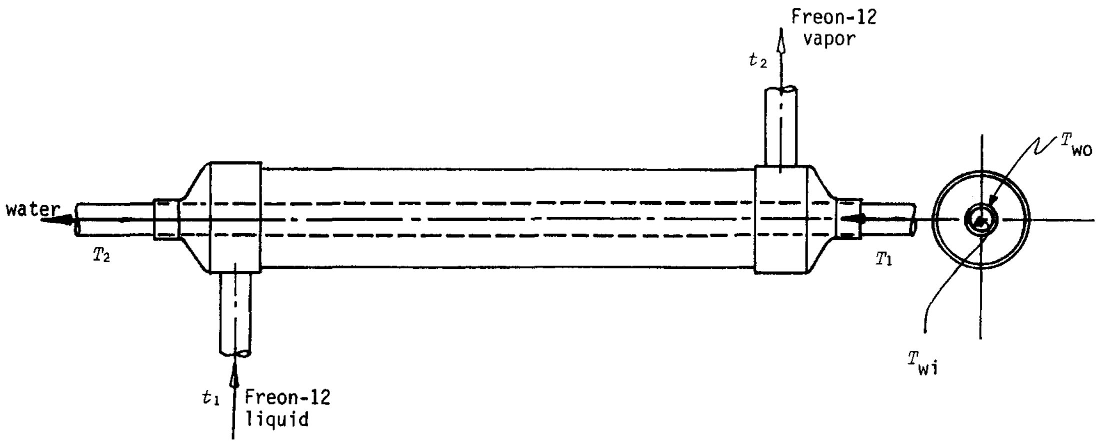

Figure 2 Schematic of the test section

Table 3 Reduced data for the isothermal cooling experiments

Test number	$\bar{h}_L$ [W/(m²−K)]	T_{wi} (°C)	$\bar{h}_0$ [W/(m²−K)]	$\overline{Nu}_D$	Re	Pr
1	6,140	7.31	177,000	93.0	11,200	4.342
2	5,130	1.76	44,300	82.0	10,300	7.504
3	6,890	11.1	68,900	105	11,800	4.637
4	7,900	10.3	29,500	124	11,500	6.296
5	6,070	12.1	56,800	91.5	13,100	4.107
6	8,120	14.0	87,100	122	13,600	3.955
7	6,200	7.40	58,100	94.1	13,400	4.365
8	7,130	4.69	25,700	113	12,600	6.971
9	8,520	12.0	17,600	134	13,500	6.124
10	7,460	11.7	74,500	114	14,300	4.590
11	6,200	10.2	57,700	93.6	14,500	4.203
12	9,460	24.9	8,180	146	14,100	5.250
13	10,300	19.0	13,600	159	14,800	5.491
14	11,800	19.6	10,400	183	15,000	5.634
15	6,930	14.4	25,400	104	16,000	4.407
16	7,100	14.0	25,900	107	17,300	4.151
17	8,450	18.2	32,000	127	18,300	3.909
18	9,770	17.6	20,600	149	18,100	4.548
19	13,100	15.5	11,500	206	17,600	6.249
20	7,180	14.5	26,100	108	18,600	4.011
21	8,220	17.9	16,800	124	19,900	4.103
22	7,870	11.5	28,800	119	20,700	4.299
23	13,000	19.5	11,300	202	20,600	5.648
24	10,800	18.0	22,800	165	22,000	4.583
25	9,510	22.0	12,300	143	22,300	3.960
26	13,900	21.3	12,100	216	22,400	5.480
27	9,920	21.6	12,900	149	23,100	4.059
28	10,100	22.3	13,100	152	25,800	3.958
29	12,200	25.3	10,500	183	26,700	4.028
30	13,900	21.5	18,500	211	27,300	4.560
31	12,500	26.0	10,800	188	29,300	3.945
32	12,600	25.5	10,800	190	31,200	4.028

$(43.40+36.75)/2=40.1°C$. The properties of water at this temperature are

$$\rho=992\,\text{kg/m}^3 \qquad \nu=6.58\times10^{-7}\,\text{m}^2/\text{s}$$

$$c_p=4180\,\text{J/(kg-K)} \qquad k=0.629\,\text{W/(m-K)}$$

Also, $\alpha=k/\rho c_p=1.517\times10^{-7}\,\text{m}^2/\text{s}$ and $\text{Pr}=\nu/\alpha=4.342$. The tube dimensions are OD = 0.5 in = 0.0127 m, ID = 3/8 in = 0.00953 m, and $L=10$ in = 0.254 m. Substituting into Equation 6 gives

$$q=992(5.52\times10^{-5})(4180)(43.40-36.75)=1520\,\text{W} \tag{11}$$

Solving Equation 7 for U_0 gives

$$U_0=\frac{1520\ln[(43.40-5.75)/(36.75-5.75)]}{\pi(0.0127)(0.254)(43.40-36.75)}=4380\,\text{W/(m}^2\text{-K)} \tag{12}$$

Applying Equation 5 gives

$$1520=\bar{h}_L\pi(0.00953)(0.254)\frac{43.40-36.75}{\ln[(43.40-T_{wi})/(36.75-T_{wi})]}$$

$$\bar{h}_L=3.01\times10^4\ln[(43.40-T_{wi})/(36.75-T_{wi})] \tag{13}$$

With the thermal conductivity of copper taken as 384 W/(m-K), Equation 9 becomes

$$T_{wi}=5.75+(1.50\times10^5)(4.74\times10^{-6}+1/\bar{h}_0) \tag{14}$$

Finally, Equation 8 gives

$$1/\bar{h}_0=2.236\times10^{-4}-1.33/\bar{h}_L \tag{15}$$

Equations 13–14 were solved simultaneously for all 32 runs of this study. The iteration scheme is as follows: assume a value for $\bar{h}_0$; calculate T_{wi} with Equation 14; calculate $\bar{h}_L$ with Equation 13; calculate an improved value for $\bar{h}_0$ with Equation 15. The equations are sensitive to small changes in $\bar{h}_0$, and so this method did not always converge. Successive iterations yielded new values that diverged in both directions from the correct value. Randomly selected values for $\bar{h}_0$ were thus chosen until this pattern emerged. Convergence was achieved when successive trials yielded a change in T_{wi} that was less than 1%. For this example, the values that satisfy the equations are

$$\bar{h}_L=6140\,\text{W/(m}^2\text{-K)},\quad \bar{h}_0=177{,}000\,\text{W/(m}^2\text{-K)},\quad T_{wi}=7.31°\text{C}$$

The values of $\bar{h}_L$ and T_{wi} were determined for the data of this study and are shown in Table 3. Also shown are the corresponding Nusselt, Reynolds, and Prandtl numbers. The hydrodynamic entrance length will be the shortest for the lowest Reynolds number. Thus

$$z_h=4.4D(\text{Re})^{1/6}=4.4(0.00953)(10{,}250)^{1/6}=0.195\,\text{m}$$

For the largest Reynolds number case, $z_h=0.235$ m. For most of the runs of this study, the flow was developed hydrodynamically before reaching the test section. The thermal entrance length was calculated with Equation 10b and ranged from 40 to 65 m. The test section is only 0.254 m long, so the flow was developing thermally.

Performing a statistical analysis of the reduced data gave the following equation for the Nusselt number:

$$\overline{\text{Nu}}_D=0.00565\,\text{Re}^{0.904}\text{Pr}^{0.812} \tag{16}$$

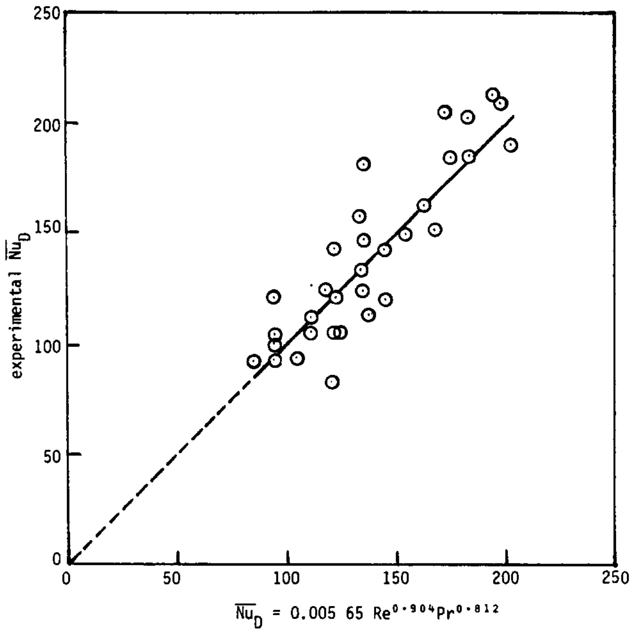

Figure 3 Comparison of experimental Nusselt number with that calculated by Equation 16

with a correlation coefficient of 0.877 and a standard deviation of 19.01. Figure 3 is a graph of Equation 16 with data points shown.

For purposes of comparison, appropriate parameters were substituted into Nusselt's equation (Equation 4). The results are provided in Table 4 and compared with Equation 16. Nusselt's equation overpredicts the Nusselt number obtained from Equation 16 by an amount that ranges from 10 to 25%.

Table 4 Comparison of the Nusselt numbers for the isothermal cooling experiments

Test number	① experimental $\overline{Nu}_D$	② Equation 16 $\overline{Nu}_D$	Equation 4 $\overline{Nu}_D$	$\frac{① - ②}{①} \times 100\%$
1	93.0	85.5	122	8.03
2	82.0	123	136	−49.9
3	105	94.4	130	10.1
4	124	119	141	4.45
5	91.5	94.2	136	−3.03
6	122	94.2	138	22.8
7	94.1	101	141	−7.2
8	113	140	157	−23.6
9	134	133	159	0.106
10	114	112	152	1.73
11	93.6	105	149	−12.3
12	146	123	156	16.0
13	159	133	165	16.2
14	183	137	168	24.8
15	104	116	159	−6.9
16	107	122	171	−14.1
17	127	122	175	3.54
18	149	137	182	7.72
19	206	173	198	16.1
20	108	127	178	−17.4
21	124	137	190	−10.6
22	119	148	199	−23.9
23	202	183	217	9.30
24	175	164	213	0.160
25	143	148	205	−3.20
26	216	225	230	10.4
27	149	156	213	−4.28
28	152	168	230	−10.6
29	183	176	239	3.78
30	211	199	253	2.62
31	188	189	255	−0.32
32	190	203	270	−6.68

Conclusions

The determination of an equation for the Nusselt number to describe the isothermal cooling of water in turbulent, thermally developing flow was the subject of this study. Data were taken, and it was found that the Nusselt number could be predicted by

$$\overline{Nu}_D = 0.00565\, Re^{0.904} Pr^{0.812}$$

over the ranges $10{,}000 < Re < 31{,}000$, $3.9 < Pr < 7.5$, and $L/D = 26.7$ with a correlation coefficient of 0.877 and a standard deviation of 19.0.

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